Distributed slack bus model formulation for the Holomorphic Embedding Load flow Method (HELM)

Openmod Lightening Talk
Via Zoom
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## Power Flow

<table>
<thead>
<tr>
<th>Variables</th>
<th>Slack Bus</th>
<th>PQ Buses</th>
<th>PV Buses (Generators)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Known</td>
<td>$</td>
<td>V</td>
<td>; \theta$</td>
</tr>
<tr>
<td>Unknown</td>
<td>$P; Q$</td>
<td>$</td>
<td>V</td>
</tr>
</tbody>
</table>

Total active power load is known, but the power losses are unknown.
Distributed slack bus model

This model states that the total power losses must be distributed amongst a group of buses and not just the slack bus.

Distribution based on participation factor:

\[
\sum_{i=1}^{N} F_i = 1 \\
F_i = \begin{cases} 
\frac{P_{gi}}{\sum_{k\in e} P_{gk}}, & \text{if } i \in e \\
0, & \text{if } i \notin e 
\end{cases}
\]

\(e \rightarrow \) Set of PV buses and the slack bus.

\(P_{gi} \rightarrow \) Scheduled active power generation at bus \(i\).

\[P_{g\text{slack}} = \sum_{i=1}^{N} P_{di} - \sum_{i=1}^{N} P_{gi}, \quad i \neq \text{slack}\]

\(P_{loss} \rightarrow \) New unknown variable
<table>
<thead>
<tr>
<th>Bus</th>
<th>Original equation</th>
<th>Embedded equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slack</td>
<td>( V_i = V_i^{sp} \angle 0^\circ )</td>
<td>( V_i(s) = 1 + (V_i^{sp} - 1)s )</td>
</tr>
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<td>PQ</td>
<td>[ \sum_{k=1}^{N} Y_{ik} V_k = \frac{S_i^<em>}{V_i^</em>} ]</td>
<td>[ \sum_{k=1}^{N} Y_{ik \text{ series}} V_k(s) = s \frac{S_i^<em>}{V_i^</em>(s^*)} - s Y_{ishunt} V_i(s) ]</td>
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<td>PV</td>
<td>[ \sum_{k=1}^{N} Y_{ik} V_k = \frac{S_i^<em>}{V_i^</em>} ]</td>
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<td>(</td>
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\[ |V_i| = V_i^{sp} \]

\[ V_i(s) * V_i^*(s^*) = 1 + \left((V_i^{sp})^2 - 1\right) s \]
New PV equation preserving the algebraic nature of the problem:

\[ \sum_{k=1}^{N} Y_{ik\,trans} \, V_k(s) = \frac{s \, P_i + F_i \, Ploss(s) - j \, Q_i(s)}{V_i^*(s^*)} - s \, Y_{i\,shunt} \, V_i(s) \]

Power series expansion: \[ V_i(s) = \sum_{n=0}^{\infty} V_i[n] \, s^n = V_i[0] + V_i[1] \, s + V_i[2] \, s^2 + \ldots \]

Equation to calculate the \( n^{th} \) term of the power series:

\[ \sum_{k=1}^{N} Y_{ik\,trans} \, V_k[n] - F_i \, Ploss[n] + j \, Q_i[n] \]

\[ = P_i \, W_i^*[n - 1] + F_i \left( \sum_{x=1}^{n-1} Ploss[x] \, W_i^*[n - x] \right) - j \left( \sum_{x=1}^{n-1} Q_i[x] \, W_i^*[n - x] \right) - Y_{i\,shunt} \, V_i[n - 1] \]
New PV equation preserving the algebraic nature of the problem:

\[
\sum_{k=1}^{N} Y_{ik\text{trans}} V_k(s) = \frac{s P_i + F_i Ploss(s) - j Q_i(s)}{V_i^*(s^*)} - s Y_{is\text{hunt}} V_i(s)
\]

Power series expansion:

\[
V_i(s) = \sum_{n=0}^{\infty} V_i[n] s^n = V_i[0] + V_i[1] s + V_i[2] s^2 + \ldots
\]

Equation to calculate the \(n^{th}\) term of the power series:

\[
\sum_{k=1}^{N} Y_{ik\text{trans}} V_k[n] - F_i Ploss[n] + j Q_i[n] = \left( P_i W_i^*[n - 1] + F_i \left( \sum_{x=1}^{n-1} Ploss[x] W_i^*[n-x] \right) \right) - j \left( \sum_{x=1}^{n-1} Q_i[x] W_i^*[n-x] \right) - Y_{is\text{hunt}} V_i[n - 1]
\]
Unbalance the equation system

New unknown variable: $P_{loss}$

New constant: $P_{g\ slack}$

Three bus system:
1. Slack bus
2. PQ bus
3. PV bus

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
G_{21} & -B_{21} & G_{22} & -B_{22} & 0 & -B_{23} & 0 & 0 & 0 \\
B_{21} & G_{21} & B_{22} & G_{22} & 0 & G_{23} & 0 & 0 & 0 \\
G_{31} & -B_{31} & G_{32} & -B_{32} & 0 & -B_{33} & -F_3 & 0 & 0 \\
B_{31} & G_{31} & B_{32} & G_{32} & 1 & G_{33} & 0 & 0 & 0 \\
G_{slack1} & -B_{slack1} & G_{slack2} & -B_{slack2} & 0 & -B_{slack3} & -F_{slack} & 0 & 0 \\
B_{slack1} & G_{slack1} & B_{slack2} & G_{slack2} & 0 & G_{slack3} & 0 & 0 & 1 \\
\end{bmatrix} \begin{bmatrix}
V_{1 re}[n] \\
V_{1 im}[n] \\
V_{2 re}[n] \\
V_{2 im}[n] \\
V_{3 re}[n] \\
V_{3 im}[n] \\
P_{loss}[n] \\
Q_{slack}[n] \\
\end{bmatrix} = \begin{bmatrix} \text{known} \end{bmatrix}$$
### Losses calculated by the classic and distributed slack bus model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Losses (MW)</th>
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<tbody>
<tr>
<td>Classic slack bus</td>
<td>1672.14</td>
</tr>
<tr>
<td>Distributed slack bus</td>
<td>1660.83</td>
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<tr>
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<td>2802.73</td>
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<tr>
<td>Distributed slack bus</td>
<td>2755.98</td>
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</tbody>
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### Coefficients calculated by classic and distributed slack bus HELM vs. tolerance

#### case1354pegase grid

<table>
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<tr>
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<tbody>
<tr>
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#### case2869pegase grid

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References


